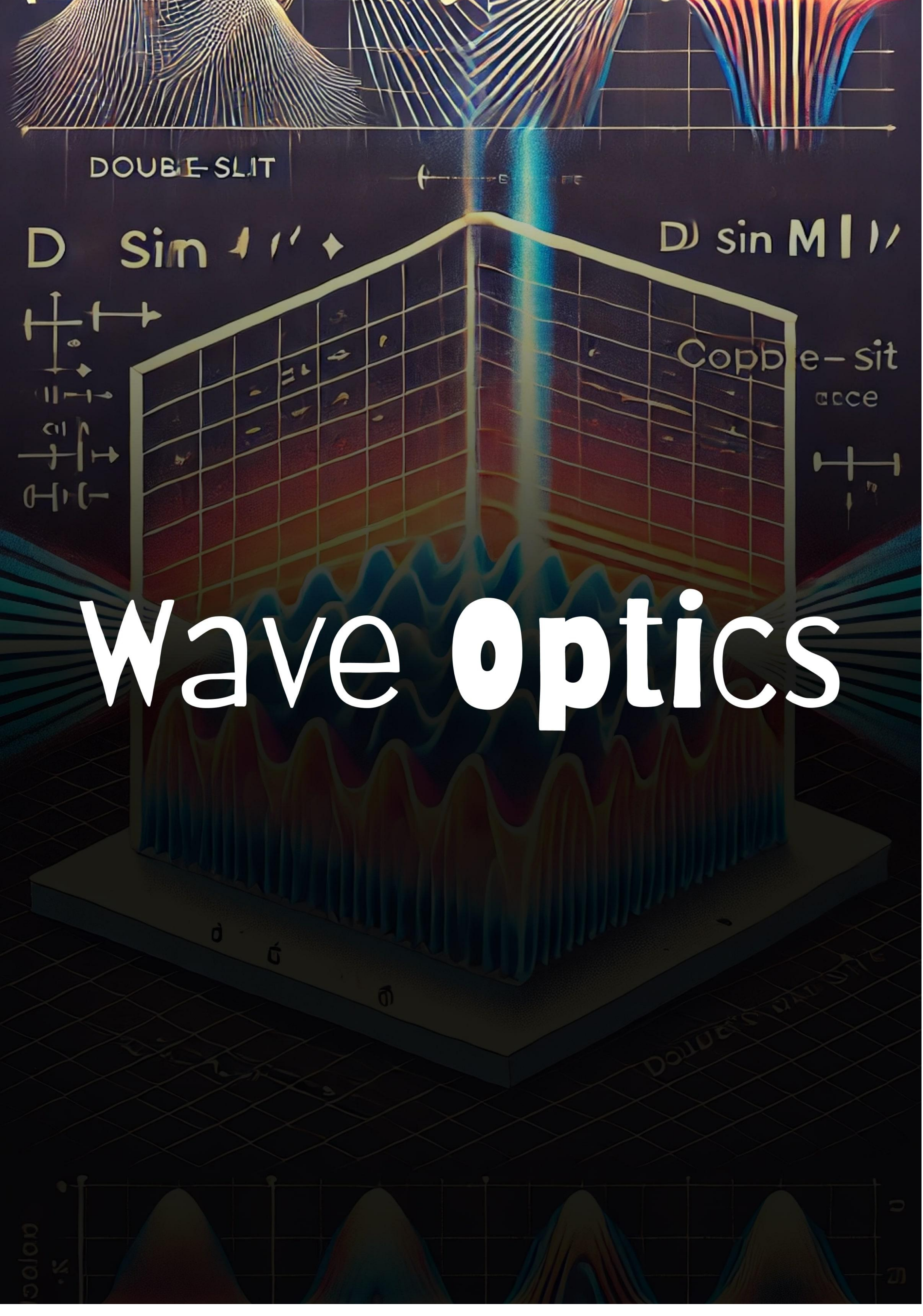
Wave Optics



Wave Optics

Equation of wave 1 = $A_1 \cos(kx - \omega t + \phi_1)$

Equation of wave 2 = $A_2 \cos(kx - \omega t + \phi_2)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\Delta\phi)}$$

↓
resultant amplitude

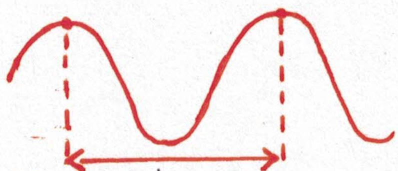
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\Delta\phi)$$

↓
resultant intensity

$$I \propto (A)^2$$

Crest to crest → phase diff = 2π

phase = argument of Cos or Sin



$\Delta\phi = 2\pi$

$\Delta x = \lambda$

$$\left(\Delta\phi = \frac{2\pi}{\lambda} \cdot \Delta x\right)$$

COHERENT SOURCES

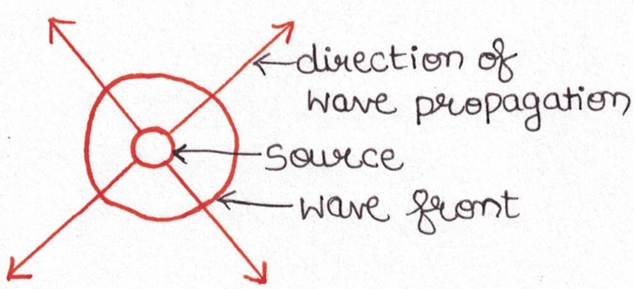
The sources which have a constant phase difference & same angular frequency.

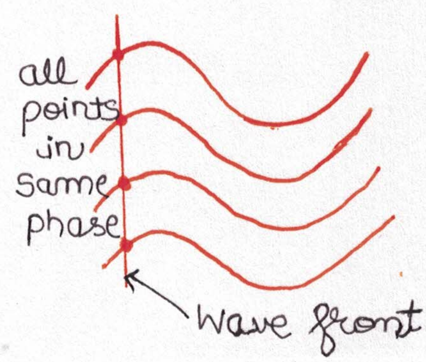
For observing inference in two light waves, it is necessary that the two sources should be coherent.

WAVE FRONT

A surface connecting points having oscillation in same phase.

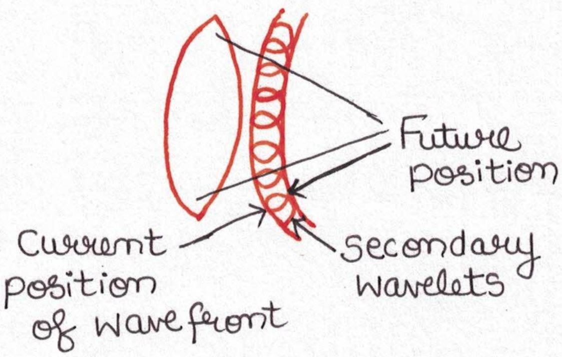
A wave front is ⊥ to the direction of propagation of wave.

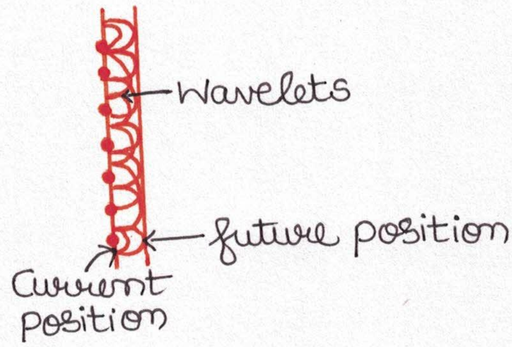




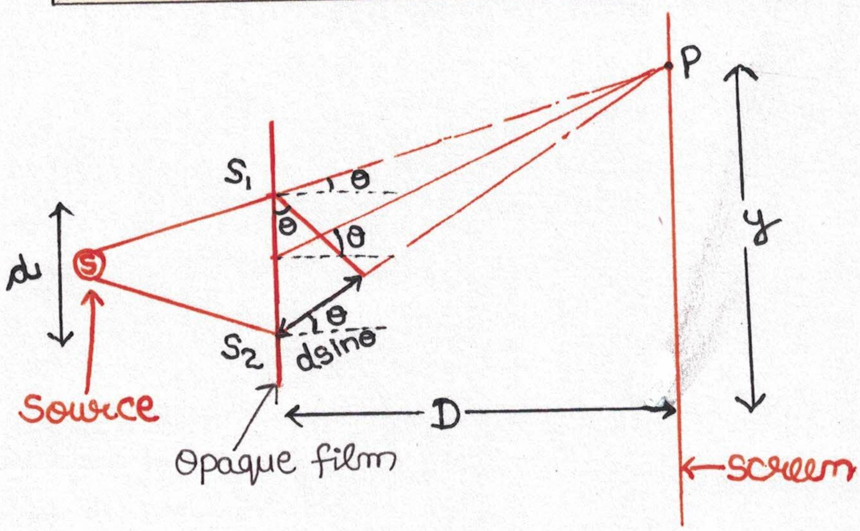
HUYGEN'S PRINCIPLE

Each point on a wavefront acts as a source of secondary wavelets travelling in all directions.
The future position of wavefront is given by the surface which is tangent to all the secondary wavelets.





YOUNG'S DOUBLE SPLIT EXPERIMENT : YDSE



For the shown YDSE setup,
(i) locate the maximas
(ii) locate the minimas
(iii) Calculate the fringe width(β)
Given that $D >> d$

★ Fringe width : Maxima to Maxima distance

$$\Delta x = d\sin\theta$$

If θ is small, then $\sin\theta \simeq \tan\theta = \frac{y}{D}$

$$\Delta x = \frac{dy}{D}$$

For maxima

$$\Delta\phi = 2n\pi$$

$$\frac{2\pi}{\lambda}\Delta x = 2m\pi$$

or $\Delta x = \left(m+\frac{1}{2}\right)\lambda$

LOCATION OF MAXIMAS

$$\frac{dy}{D} = m\lambda$$

or $\left(y = m\lambda\frac{D}{d}\right)$

$$y = \underset{\downarrow\ \text{central}}{0},\ \underset{\searrow 1^{st}}{\frac{\lambda D}{d}},\ \frac{2\lambda D}{d}\ \text{--------}\ \Big|\ \underset{\searrow 1^{st}}{-\frac{\lambda D}{d}},\ -\frac{2\lambda D}{d}\ \text{-------}$$

$|m| \rightarrow$ order of maxima

LOCATION OF MINIMAS

$$\frac{dy}{D} = \left(m+\frac{1}{2}\right)\lambda$$

or $\left(y = \left(m+\frac{1}{2}\right)\frac{\lambda D}{d}\right)$

$$y = \underset{\searrow 1^{st}}{\frac{\lambda D}{2d}},\ \underset{\searrow 2^{nd}}{\frac{3\lambda D}{2d}}\ \text{--------}\ \Big|\ \underset{\searrow 1^{st}}{-\frac{\lambda D}{2d}},\ \underset{\searrow 2^{nd}}{-\frac{3\lambda D}{2d}}\ \text{-------}$$

for +ve side, $(m+1) \rightarrow$ order of minimum

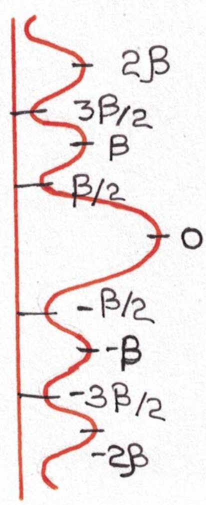
for −ve side, $|m| \rightarrow$ order of minimum

FRINGE WIDTH

$$\beta = y_1 - y_0$$

$$\left(\beta = \frac{\lambda D}{d}\right)$$

Pattern observed ⇒



Que.) In a YDSE the slit separation is 0.2mm & screen distance is 1m. 3rd bright fringe is at a distance of 7.5mm from the central fringe, Find wavelength of light.

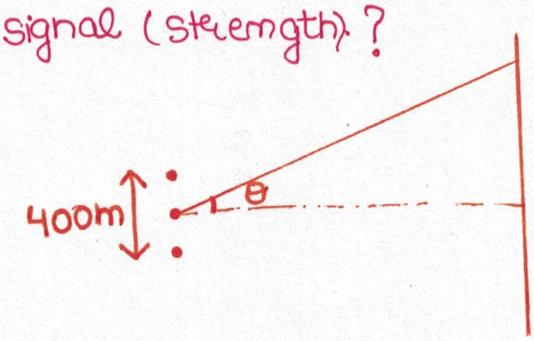
$$3\beta = 7.5mm$$

$$\lambda\left(\frac{10^3}{0.2}\right) = \frac{7.5}{3}mm$$

$$\lambda = \frac{75}{30} \times \frac{2}{10^4} = 5\times10^{-4}mm$$

$$\lambda = 500nm$$

Que.) Distance b/w two antennas of a radio-station is 400m. The station operates at 1500 KHz. At distances must greater than 400m, in which directions will you receive best signal (strength)?



$$\nu = 1500\ KHz$$

$$\lambda = \frac{3\times10^8}{1.5\times10^6}$$

$$\lambda = 200m$$

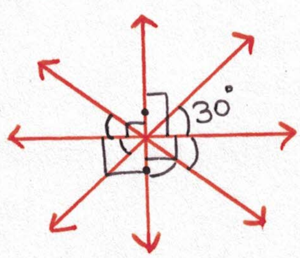
$$\Delta x = m\lambda$$

$$d\sin\theta = m\lambda$$

$$\sin\theta = m\left(\frac{200}{400}\right) = \frac{m}{2}$$

$$\theta = \sin^{-1}\left(\frac{m}{2}\right)$$

$$\therefore \sin\theta = -1, -\frac{1}{2}, 0, +\frac{1}{2}, +1$$



Total 8 directions

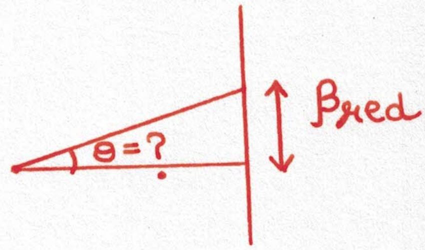
YDSE WITH WHITE LIGHT

Here b/w the central maximum and the 1st maximum of red, a VIBGYOR will be observed with central maximum being white.

However after going a little further, there will be a confused pattern of colors & if we go even further, almost a white illumination will be observed because at that point, some shade of each color will be having a maximum.

Que:) White light illuminates two slits separated by 0.1mm. Calculate angular width in degrees of the 1st full color visible spectrum on either sides of the central maximum.

$$(0.1mm)\sin\theta = (700\times10^{-9}\times10^{3}mm)$$



$$\sin\theta = 7\times10^{-4}$$

$$\theta = \sin^{-1}(0.007)$$

$$\simeq 0.4010^{\circ}$$

$$\tan\theta = \frac{\beta}{D} = \frac{\lambda}{d} \simeq \theta$$

Que:) Locate the 1st point on the screen from central maximum where the intensity is half that at central max., slits being identical.

at center, $I = 4I_0$

at point, $I = I_0 + I_0 + 2I_0\cos\Delta\phi$

If slits are identical,

$$I_{at\,any\,pt} = I_{at\,centre}\cos^2\left(\frac{\Delta\phi}{2}\right)$$

Here, $I = I_c\cos^2\left(\frac{\Delta\phi}{2}\right)$

$$\frac{1}{2} = \cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$\frac{\Delta\phi}{2} = 45^{\circ}$$

$$\Delta\phi = \frac{\pi}{2} = \frac{2\pi}{\lambda}.\Delta x$$

$$\Delta x = \frac{\lambda}{4}$$

$$y = \frac{\lambda D}{4d}$$

$$y = \beta/4$$

Que.) Where will I be $1/4^{th}$ of $I_{at\ centre}$.

$$\cos\left(\frac{\Delta\phi}{2}\right) = \frac{1}{4}$$

$$\Delta\phi = \frac{2\pi}{3} = \frac{2\pi}{\lambda}\cdot\Delta x$$

$$\Delta x = \frac{\lambda}{3}$$

$$y = \frac{\lambda D}{3d} = \beta/3$$

(Slit Intensity $\propto$ Slit width)

RELATION B/W VELOCITY OF LIGHT & REFRACTIVE INDEX

When a wave travels from one medium to another, its frequency does not change.

$$\lambda\nu = v$$

$$\left(\mu = \frac{c}{v}\right) = \frac{\text{Speed in air}}{\text{Speed in medium}}$$

air $\frac{c}{\lambda} \to \frac{v}{\lambda'}$ medium

$$c = \lambda\nu \quad , \quad v = \lambda'\nu$$

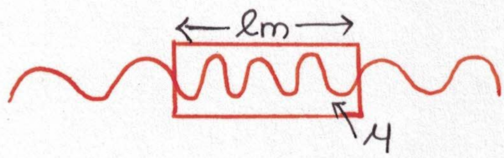
$$\left(\lambda_{medium} = \frac{\lambda_{air}}{\mu}\right)$$

AIR EQUIVALENT PATH

The equivalent path required in air to produce the same phase difference as a given thickness of material.

$$l_{air} = \frac{\lambda}{\lambda'_{max}} \; l_m$$

$$\left(l_{air} = \mu\, l_{medium}\right)$$



★ To complete same no. of cycles in air, the wave would have travelled a distance greater than l.

MEDIUM EQUIVALENT PATH

The path length required in medium 2 to produce the same phase difference as a given thickness of medium 1, is called the M.E.P. in medium 2.

From the previous discussion, we can say that

$$\left(l_2 = \frac{\mu_1}{\mu_2}\cdot l_1\right)$$

Que.) What will happen to the fringe width if a YDSE setup was immersed in a medium of refractive index 'μ'?

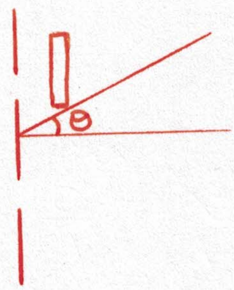
$$\beta_m = \lambda_m \frac{D}{d}$$

$$= \frac{\lambda D}{\mu d}$$

$$= \frac{\beta}{\mu}$$

ie pattern will shrink by a factor of 'μ'.

Que.) Calculate the shift in central maximum if a film of thickness 't' & refractive index 'μ' is placed in front of one of the slits.



by definition of central max.

$$(S_1P)_{optical} = (S_2P)_{optical}$$

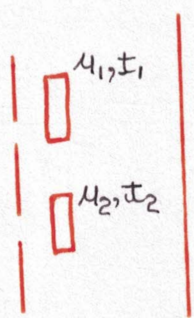
$$(S_1P)_{geometrical} + \mu t - t = (S_2P)_{geometrical}$$

$$-d\sin\theta + (\mu-1)t = 0$$

$$(\mu-1)t = d\sin\theta \simeq \frac{dy}{D}$$

(Fringe shift) $y = (\mu-1)t.\frac{D}{d}$

Que.) Calculate fringe shift & condition for zero shift.



$$(S_1P)_{opt} = (S_2P)_{opt}$$

$$(S_1P)_{geo} + \mu_1 t_1 - t_1 = (S_2P)_{geo} + \mu_2 t_2 - t_2$$

$$(\mu_1-1)t_1 - (\mu_2-1)t_2 = d\sin\theta \simeq \frac{dy}{D}$$

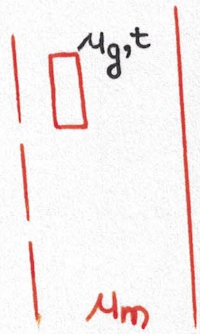
$$y = \left[(\mu_1-1)t_1 - (\mu_2-1)t_2\right]\frac{D}{d}$$

For zero shift,

$$\mu_1 t_1 - t_1 = \mu_2 t_2 - t_2$$

$$(\mu_1-1)t_1 = (\mu_2-1)t_2$$

Que.) Wavelength in air $= \lambda$
whole system is placed in medium of refractive index 'μ_m'.



Water eq. path diff $= 0$

$$(S_1P)_{\text{water eq.}} = (S_2P)_{\text{water eq.}}$$

$$(S_1P)_{geo} + \frac{\mu_g}{\mu_m} \cdot t - t = (S_2P)$$

$$\frac{(\mu_g - \mu_m)}{\mu_m} t = \frac{dy}{D}$$

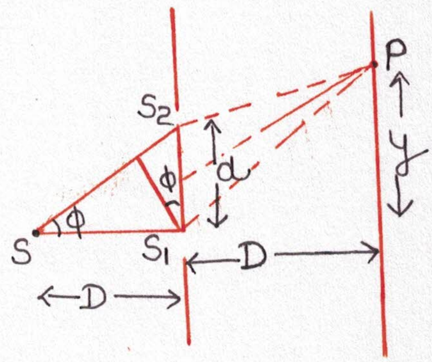
$$\text{Fringe shift } y = (\underbrace{\mu_{gm}}_{\mu_{relative}} - 1)\, t \cdot \frac{D}{d}$$

$$\beta = \frac{\lambda' D}{d}$$

$$\beta = \frac{\lambda D}{\mu_m d}$$

YDSE WITH ASYMMETRIC SOURCE LOCATION

Que.) Locate central maximum if source is located directly in front of one of the slits.



$$SS_2 - SS_1 = d\sin\phi = d\tan\phi$$

$$= d \cdot \frac{d}{D}$$

$$= \frac{d^2}{D}$$

$$S_2P - S_1P = \frac{dy}{D}$$

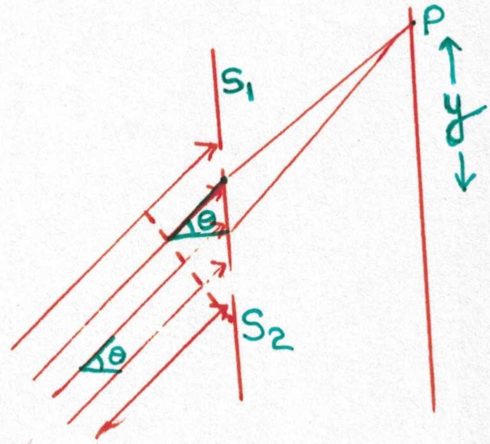
$$(SS_2 - SS_1) + (S_2P - S_1P) = 0$$

$$\frac{dy}{D} - \frac{d^2}{D} = 0$$

$$\frac{d}{D}(y - d) = 0$$

$$y = d$$

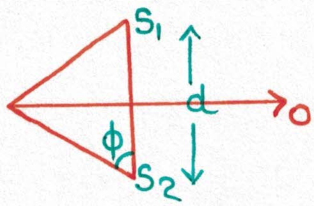
Que.) Locate central maximum.



$$d\sin\theta - d\sin\phi = 0$$

$$\sin\phi = \frac{y}{D}$$

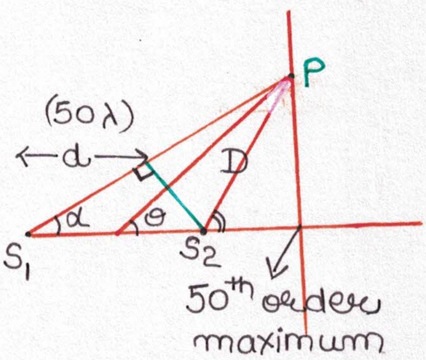
$$y = D\sin\phi$$



COUNTING TOTAL NO. OF MAXIMAS & MINIMAS

To count the no. of maximas and minimas, we calculate the max. possible path difference & then use the fact that every maxima will occur at a **path** difference of integer multiple of 'λ' & minima will correspond to odd multiple of '$\lambda/2$'.

Que.) Calculate total no. of maximax & minimas. Also find position of 49^{th} order maximum.



Maximas = 99

Minimas = 100

$$\Delta x = 49\lambda$$

$$d\cos\theta = 49\lambda$$

$$\cos\theta = \frac{49}{50}, \quad \sin\theta = \frac{3\sqrt{11}}{50}$$

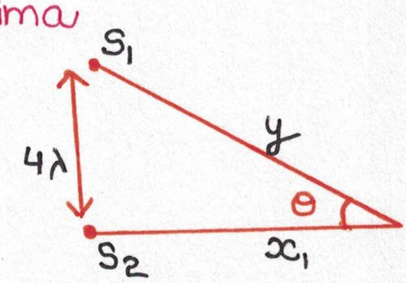
$$\tan\theta \simeq \frac{y}{D} \simeq \frac{1}{5}$$

$$y = \frac{D}{5}$$

49^{th} order maxima = path difference

$= 49\lambda$

Que.) As u move along the x-direction beginning from S_2, What is the min. value of x for which u get
(a) Maximum
(b) Minimum
(c) also tell the order of maxima/minima



$$\tan\theta = \frac{4\lambda}{x_1}$$

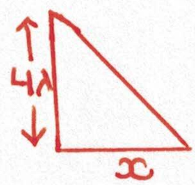
as x is ↑, path diff. is ↓
at 1st maxima, path diff. $= 3\lambda$

$$x = \frac{7}{6}\lambda$$

(a) $\because \sqrt{x^2+16\lambda^2} = x = 3\lambda$

$$\cancel{x^2} + 16\lambda^2 = \cancel{x^2} + 9\lambda^2 + 6\lambda x$$

$$x = \frac{7\lambda}{6}$$



(b) $\sqrt{x^2+16\lambda^2} = 35\lambda + x$

$$16\lambda^2 = \frac{49}{4}\lambda + 7\lambda x$$

$$x = \frac{15}{4}\cdot\frac{\lambda}{7} = \frac{15}{28}\lambda$$

(c) 3rd Maxima, 4th minima
(d) Find max & min. value of x.

For maxima,

$$x^2 + 16\lambda^2 = x^2 + \lambda^2 + 2\lambda x$$

$$x = \frac{15\lambda}{2}$$

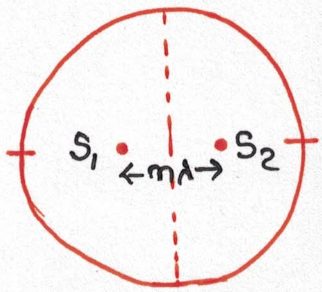
For minima,

$$x^2 + 16\lambda^2 = x^2 + \frac{\lambda^2}{4} + \lambda x$$

$$x = \frac{63}{4}$$

Que.) $R >> n\lambda$

Calculate total no. of maximas & minimas on circumference.



$2n-1 + 2n-1 + 2$

$(n-1)\lambda = 1$

$n\lambda = \lambda + N - 1$

$(n-1)\lambda + 1 - N$

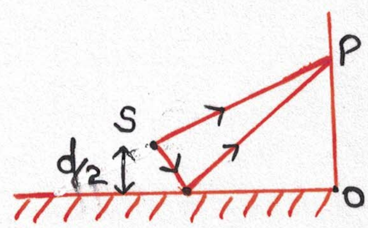
Maxima $= 4n$

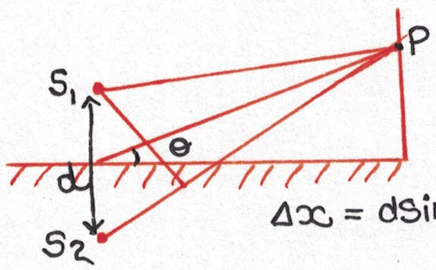
Minima $= 4[n]$

NOTE: Whenever a wave gets reflected from a denser medium (higher refractive index), it undergoes a phase shift of π.
This must be taken into account while considering interference of reflected waves.

Que.) For Lloyd's mirror setup, locate the maxima & minima.

S_2P is π shifted due to reflection





$\Delta x = d\sin\theta$

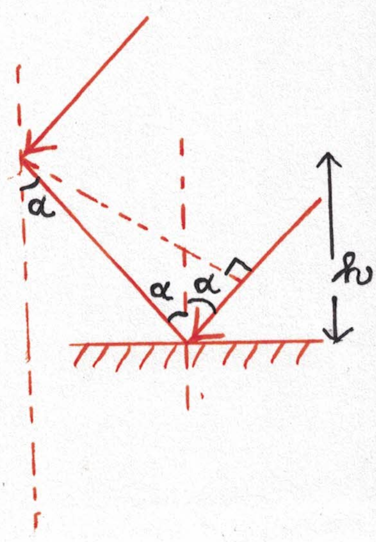
$\therefore$ Crest of S_1 overlaps with trough of S_2.
Conditions for maxima & minima interchange & at O, minima obtained.

Que.)

$\Delta x = h\sec\alpha\,(1+\cos 2\alpha)$

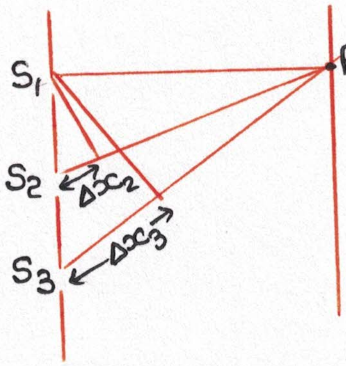
$\frac{\lambda}{2}(2n+1) = 2h\cos\alpha$

$h = \frac{\lambda(2n+1)}{4\cos\alpha}$



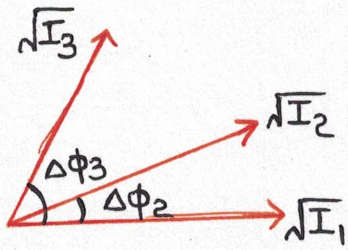
INTERFERENCE FROM MULTIPLE SLITS

Here we can find the path differences from the shortest path of all the incident light, phasors at any pt. and get the resultant intensity.

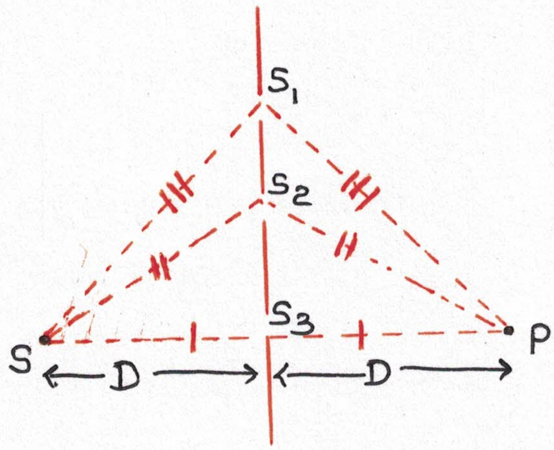


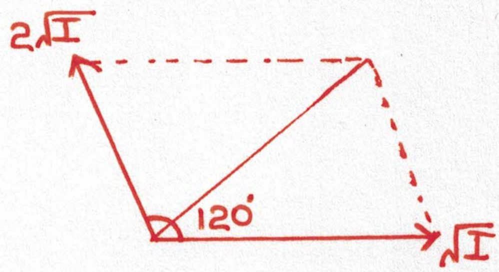
$$\Delta\phi_2 = \frac{2\pi}{\lambda} \cdot \Delta x_2$$

$$\Delta\phi_3 = \frac{2\pi \cdot \Delta x_3}{\lambda}$$



Que.)





$$S_1P - S_2P = \lambda/6$$

$$S_1P - S_3P = 2\lambda/3$$

$$\Delta\phi_1 = \frac{2\pi}{\lambda} \cdot \frac{2\lambda}{6} = \frac{2\pi}{3}$$

$$\Delta\phi_2 = \frac{2\pi}{\lambda} \cdot \frac{4\lambda}{3} = \frac{8\pi}{3}$$

$$I_R = 3I$$

$$I_R = \left[\sqrt{(I)^2 + (2\sqrt{I})^2 + 2(\sqrt{I})(2\sqrt{I}).\cos 120^\circ}\right]^2$$

Que.) In a YDSE, intensity due to one slit = I_1 & due to other is I_2. Find ratio of max. to min. intensity on screen.

$$I_R = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$$

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$$

$$9 = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_C}-\sqrt{I_F}}\right)^2 \quad \text{, let } \sqrt{\frac{I_C}{I_F}} = x$$

$$\frac{x+1}{x-1} = 3$$

$$x = 2 \Rightarrow I_C = 4I_1$$

$$I_1 = I_C\cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$\cos\left(\frac{\Delta\phi}{2}\right) = \sqrt{\frac{I_1}{I_C}} = \frac{1}{2}$$

or $\left(\frac{\Delta\phi}{2}\right)_{least} = 60^\circ$

$$\Delta\phi = \frac{2\pi}{3} = \frac{2\pi}{\lambda}\cdot\frac{dy}{D}$$

$$y = \frac{\lambda D}{3d}$$

SINGLE SLIT DIFFERENCE

Consider a point P on the screen as shown in the fig. If we want a minima at P, then the phasor sum of the various rays coming from the slit should be zero. We also know that the phasors of intermediate points will be uniformly distributed b/w the phasor of top-most point and the bottom-most point.

Thus if these two phasors are integer no. of cycles apart then a minimum will be obtained.

Mathematically, For minimum

$$\frac{2\pi}{\lambda} d\sin\theta = 2n\pi$$

$$\sin\theta = \frac{m\lambda}{d} \quad ; m = \pm 1, \pm 2 \ldots\ldots$$

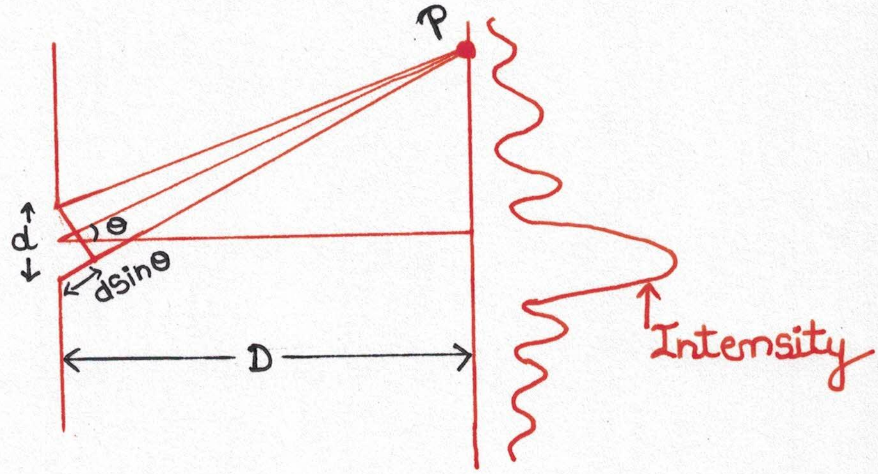
$*m \neq 0$

For maxima,

[bisector of 2 minima]

$$\frac{2\pi}{\lambda} \cdot d\sin\theta = (2m+1) \cdot \pi$$

$$\sin\theta = \frac{(2m+1)\lambda}{2d}$$



NOTE: 1.) Centre of screen will have a max. because all the phasors are superimposing in same phase.

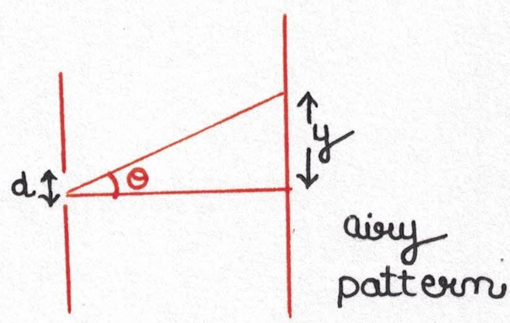
★ Width of central max. $= \frac{2\lambda}{d}$

2.) other than the central max., the other maxima are very faint.

DIFFRACTION FROM CIRCULAR APERTURE

For central bright

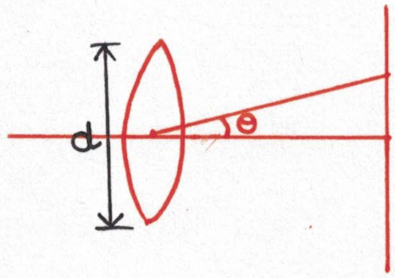
$$\theta = \frac{1.22\lambda}{d}$$



RESOLUTION OF OBJECTS

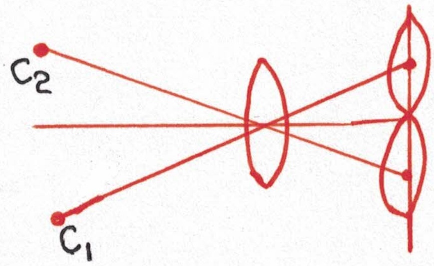
Due to diffraction effects, the image of a distant object is not a single point in the focal ~~point~~ plane but it has an angular radius of $(1.22\lambda/d)$ where d is the aperture of the lens.

$$\theta = \frac{1.22\lambda}{d}$$

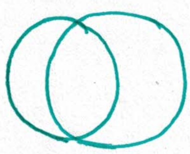


∴ To get a sharp image, the aperture of the lens should be large.

★ For resolution, angular separation b/w two objects should be $(1.22\lambda/d)$

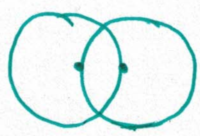


$C_1C_2 < \frac{1.22\lambda}{d}$



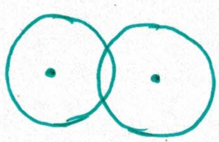
not resolved

$C_1C_2 = \frac{1.22\lambda}{d}$



barely resolved (just)

$C_1C_2 > \frac{1.22\lambda}{d}$



Clearly resolved

ie. for two objects to be clearly resolved, their angular separation must be atleast $(1.22\lambda/d)$

POLARISATION

A polarised is a thin film of some material which allows only that component of electric field to pass which is along its axis.

Thus if the angle of electric field with polaroid axis is 'θ', then the transmitted field is '$E\cos\theta$'.

$$\text{Intensity} \propto (\text{Electric field})^2$$

Thus, the transmitted intensity $\propto \langle E^2 \cos^2\theta \rangle$

i.e. $I_{trans.} \propto \frac{E^2}{2}$

whereas the incident/unpolarised light will have an intensity $\propto E^2$

Thus, if I = incident intensity on the polariser, then the transmitted intensity $= I/2$

★ If two polarising films are used, one behind the other, then the second one is known as analyser.

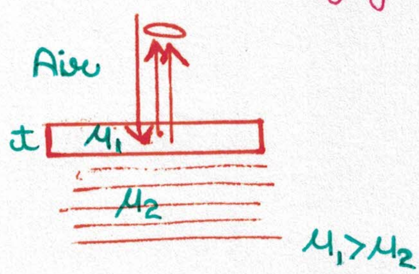
If θ = angle b/w polariser & analyser axis, then the transmitted intensity $= I\cos^2\theta$

where I = intensity after crossing the polariser.

$\left(\text{Scattering} \propto \frac{1}{\lambda^4}\right)$

INTERFERENCE FROM THIN FILMS (NEAR NORMAL INCIDENCE)

Que.) A film of thickness 't' is observed at near normal incidence. Find the wavelengths which will be strongly reflected.



For maximum,

$$2\mu t = (2n+1)\frac{\lambda}{2}$$

(because one of them is π shifted)

$$\lambda = \frac{4\mu t}{(2m+1)}$$

• When t is very large, there are many values of m to bring λ b/w 400nm & 700nm. Thus maximas of many lights will be through and so no interference will be observed due to which interference is only seen in thin films.

Que.) $\mu_1 = 1.33$, $\mu_2 = 1.5$, $\lambda = 600nm$

$$2\mu_1 t = m\lambda$$

$$t = m\lambda/2\mu_1$$

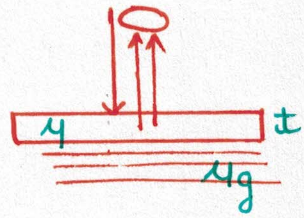
$$t = \frac{n(600)\times 3}{24} = \frac{900n}{4}$$

$$t = 225n$$

$$(t)_{min} = 225\ nm$$

Que.) non - reflecting layer $\Rightarrow$ minima

$$2\mu_1 t = (2n+1)\frac{\lambda'}{2}$$



$(1 < \mu_1 < \mu_g)$

$$t = \frac{(2n+1)\lambda'}{4\mu}$$

$$1 < \mu < 1.5$$

$$\lambda' = \mu\lambda$$

$$t = (2n+1)\frac{\lambda}{4}$$